

Section 6.5: Properties of Logarithms

$$\text{Rule 1: } \log_b (M \cdot N) = \log_b M + \log_b N$$

Examples:

$$\log_2(8 * 4) = \log_2(8) + \log_2(4)$$

$$\log_3(9 * 81) = \log_3(9) + \log_3(81)$$

$$\log_7(xy) = \log_7(x) + \log_7(y)$$

$$\log_5(125) + \log_5(625) = \log_5(125 * 625)$$

$$\log_5(y) + \log_5(z) = \log_5(yz)$$

$$\text{Rule 2: } \log_b \left( \frac{M}{N} \right) = \log_b M - \log_b N$$

Examples:

$$\log_2 \left( \frac{8}{4} \right) = \log_2(8) - \log_2(4)$$

$$\log_3 \left( \frac{9}{81} \right) = \log_3(9) - \log_3(81)$$

$$\log_7 \left( \frac{x}{y} \right) = \log_7(x) - \log_7(y)$$

$$\log_5(125) - \log_5(625) = \log_5 \left( \frac{125}{625} \right)$$

$$\log_5(y) - \log_5(z) = \log_5 \left( \frac{y}{z} \right)$$

$$\text{Rule 3: } \log_b(M^k) = k \cdot \log_b M$$

Examples:

$$\log_6(36^4) = 4\log_6(36)$$

$$\log_9(81^2) = 2\log_9(81)$$

$$\log_5(x^7) = 7\log_5(x)$$

$$4\log_2(64) = \log_2(64^4)$$

$$3\log_b(x) = \log_b(x^3)$$

$$\text{Rule 4: } \log_b(1) = 0$$

Examples:

$$\log_3(1) = 0$$

$$\ln(1) = 0$$

$$\log(1) = 0$$

$$\text{Rule 5: } \log_b(b) = 1$$

Examples:

$$\log_2(2) = 1$$

$$\log_4(4) = 1$$

$$\log(10) = 1$$

$$\ln(e) = 1$$

$$\text{Rule 6: } \log_b(b^k) = k$$

Examples:

$$\log_4(4^5) = 5$$

$$\log_3(3^9) = 9$$

$$\ln(e^7) = 7$$

$$\log(10^2) = 2$$

$$\text{Rule 7: } b^{\log_b(k)} = k$$

Examples:

$$3^{\log_3(7)} = 7$$

$$5^{\log_5(3)} = 3$$

$$4^{\log_4(x)} = x$$

Where:

$b > 0$  but  $b \neq 1$ , and  $M$ ,  $N$ , and  $k$  are real numbers but  $M$  and  $N$  must be positive!

#1 - 12: Evaluate each expression without a calculator. Check your answer using your calculator.

- |                 |                   |                    |
|-----------------|-------------------|--------------------|
| 1) $\log_2 16$  | 2) $\log_3 9^2$   | 3) $\log_4 16^3$   |
| 4) $\log_7 1$   | 5) $\log_8 8^5$   | 6) $\log_6 216$    |
| 7) $\log_3 243$ | 8) $\ln(e^4)$     | 9) $\ln(e)$        |
| 10) $\ln(1)$    | 11) $\log_2 64^3$ | 12) $\log_5(25^2)$ |

#13 – 24: Expand into sums and differences of logarithms (express exponents as multiplication).

- |                                |                                     |   |
|--------------------------------|-------------------------------------|---|
| 13) $\log_3(x^2y^3)$           | 14) $\log_4(xy^3z^5)$               | 15) $\log_5(x^2y^6z)$                     |
| 16) $\log_b \frac{x^4}{y}$     | 17) $\log_2 \frac{xy^3}{z^2}$       | 18) $\log_7 \frac{x}{yz^3}$               |
| 19) $\log_2 \frac{xy}{w^2z^5}$ | 20) $\log_3(x^2y)$                  | 21) $\log_4(x^3y^4)$                      |
| 22) $\log_4(y\sqrt{z})$        | 23) $\log_2(x^2 \cdot \sqrt[3]{y})$ | 24) $\log_5 \frac{\sqrt{x}}{\sqrt[3]{y}}$ |

#25 - 36: Write the expression as a single logarithm. Write your answer with only positive exponents.

- |                                   |  |  |
|-----------------------------------|--|--|
| 25) $3\log_2 x + 4\log_2 y$       | 26) $5\ln x + \ln y + 3\ln z$          | 27) $2\log_3 x + 4\log_3 y + \log_3 z$ |
| 28) $2\log x - 3\log y$           | 29) $5\log_2 x + 3\log_2 y - \log_2 z$ | 30) $4\log x + 2\log y - 3\log z$      |
| 31) $4\log x - 2\log y - 3\log z$ | 32) $2\ln x - 4\ln y + \ln z$          | 33) $-2\log_3 x + \log_3 y + \log_3 z$ |
| 34) $-\ln x + \ln y + \ln z$      | 35) $\ln x + 3\ln y - 2\ln z$          | 36) $4\ln x - 2\ln y + 5\ln z$         |

#37 - 45: Use the change of base formula and your calculator to evaluate each logarithm, round your answer to 2 decimal places.

- |                 |                   |                    |
|-----------------|-------------------|--------------------|
| 37) $\log_2 3$  | 38) $\log_7 14$   | 39) $\log_3 5$     |
| 40) $\log_9 36$ | 41) $\log_4 0.65$ | 42) $\log_2 0.25$  |
| 43) $\log_5 18$ | 44) $\log_6 7$    | 45) $\log_9 0.123$ |

#46 – 57: Find the following. Given  $\log_b x = 5$ ,  $\log_b y = 10$ ,  $\log_b z = 7$ .

- |                            |                               |                             |
|----------------------------|-------------------------------|-----------------------------|
| 46) $\log_b x + \log_b y$  | 47) $\log_b y + \log_b z$     | 48) $\log_b x^3$            |
| 49) $\log_b y^4$           | 50) $\log_b xy$               | 51) $\log_b yz$             |
| 52) $\log_b(x^2y^3)$       | 53) $\log_b(y^3z^5)$          | 54) $\log_b(x^2y^6z)$       |
| 55) $\log_b \frac{x^4}{y}$ | 56) $\log_b \frac{xy^3}{z^2}$ | 57) $\log_b \frac{x}{yz^3}$ |